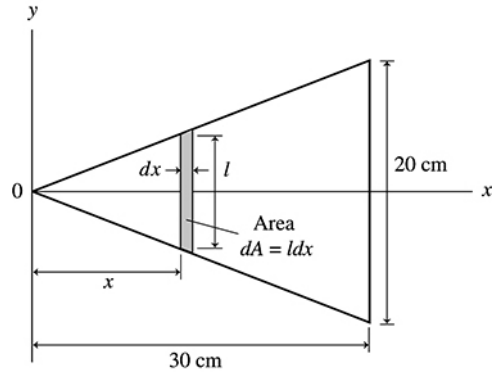
12.53. Visualize:



Solve: We will consider a vertical strip of width dx and of mass dm at a position x from the origin. The formula for the x component of the center of mass is

$$x_{\text{cm}} = \frac{1}{M}\int x\, dm$$

The area of the steel plate is $A = \frac{1}{2}(0.2\text{ m})(0.3\text{ m}) = 0.030\text{ m}^2$. Mass dm in the strip is the same fraction of M as dA is of A. Thus

$$\frac{dm}{M} = \frac{dA}{A} \Rightarrow dm = \frac{M}{A}dA = \left(\frac{0.800\text{ kg}}{0.030\text{ m}^2}\right)dA = (26.67\text{ kg/m}^2)l\,dx$$

The relationship between l and x is

$$\frac{l}{0.20\text{ m}} = \frac{x}{0.30\text{ m}} \Rightarrow l = \frac{2}{3}x$$

Therefore,

$$x_{\text{cm}} = \frac{1}{M}\int (26.67\text{ kg/m}^2)\left(\frac{2}{3}\right)x^2\,dx = \frac{(17.78\text{ kg/m}^2)}{M}\frac{x^3}{3}\bigg|_{0\text{ m}}^{0.3\text{ m}} = \frac{(17.78\text{ kg/m}^2)}{0.8\text{ kg}}\frac{(0.3\text{ m})^2}{3} = 20\text{ cm}$$

Due to symmetry $y_{\text{cm}} = 0$ cm.